




Teaching of Geometry with GeoGebra Software in High School Students of an Educational Institution in Lima

Enseñanza de la geometría con el software GeoGebra en estudiantes secundarios de una institución educativa en Lima

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Summary

The study evaluated the effects of the use of GeoGebra software in the teaching of geometry with high school students in the development of their capacities for reasoning and demonstration, mathematical communication and problem-solving. The framework of the study was the increasing presence of technologies in the school teaching of mathematics in young people of the digital age, in a context where education favors the active participation of students in the strengthening of their own abilities. The students were observed in two groups, the group intervened exposed to the use of GeoGebra software and the control group exposed to traditional teaching without the use of the software. Both groups were evaluated with a Geometry Learning Assessment Test that was applied *before* and *after* the intervention. The results suggest that the use of the GeoGebra software had effects in the strengthening of the three capacities, with improvements that were significant at high levels. Also, the scores reached at the time *afterwards* were favorable to the group intervened in the three capacities, with significant differences at moderate levels.

Keywords: Teaching geometry, GeoGebra software, educational technologies, problem-solving.

Resumen

El estudio evaluó los efectos del empleo del *software* GeoGebra en la enseñanza de la geometría con estudiantes secundarios en el desarrollo de sus capacidades para el razonamiento y demostración, la comunicación matemática y la resolución de problemas. El marco del estudio fue la presencia creciente de tecnologías en la enseñanza escolar de las matemáticas en jóvenes de la era digital, en un contexto donde la educación favorece la participación activa de los estudiantes en el fortalecimiento de sus propias capacidades. Los estudiantes fueron observados en dos grupos, el grupo intervenido expuesto al empleo del *software* GeoGebra y el grupo de comparación expuesto a una enseñanza tradicional sin el empleo del *software*. Ambos grupos fueron evaluados con una Prueba de Evaluación del Aprendizaje en Geometría que se aplicó en momentos *antes* y *después* de la intervención. Los resultados sugieren que el empleo del *software* GeoGebra tuvo efectos en el fortalecimiento de las tres capacidades, con mejoras que resultaron significativas a niveles altos. También que las puntuaciones alcanzadas en el momento *después* fueron favorables al grupo intervenido en las tres capacidades, con diferencias significativas a niveles moderados.

Palabras clave: Enseñanza de la geometría, *software* GeoGebra, tecnologías educativas, resolución de problemas.

Introduction

The use of technologies such as interactive boards, tablets, smart phones or software, which facilitate the learning process of specific fields, has a growing presence in the school teaching of mathematics in children and young people of the digital age (Korenova, 2017). This is in line with current educational trends that promote learning through an active participation of the students to strengthen their own capacities (Hernández & Villalba, 2001). In addition, a growing number of studies on the effects of the ICT use in the classroom associate them with the development of collaborative learning environments, improvement in motivation or interest, a greater tendency towards research, or the strengthening of intellectual skills such as reasoning and problem-solving (De la Chica, 2010). Other studies (Beeland, 2002; Weaver, 2000) associate them with a better performance and favorable attitudes towards mathematics.

Apparently, the educational software incorporation in the teaching of mathematics -and geometry in particular-, is a need that must begin to be covered in the short term. However, a change like this, which can only occur under optimal conditions if substantive changes are made simultaneously in the curriculum, is also perceived as a problem (Silva, Gros, Garrido & Rodríguez, 2006; Hernández, 2006). It is considered that its implementation would imply re-signifying and re-learning processes and ways of work that the teachers know and master and that are deeply rooted in their daily practices (Sepúlveda & Calderón, 2007).

On the other hand, teacher training has not been permeable to the signals emitted by these needs for change nor to the evidence that the research reports. The curricula continue to be full of pedagogical theory, disciplinary contents, didactics or evaluation methodologies, thus leaving marginal spaces for the development of skills that would allow the use of technological tools in the teaching process (Silva, Gros, Garrido & Rodríguez, 2006). In addition, the few technological training opportunities for active teachers are offered in the form of limited workshops that basically focus on providing general

information about educational software, and on some information about the basic skills needed for its use, without addressing the essential issue of how to integrate it into the teaching process (Parsad, Lewis & Farris, 2001). Thus, the technical training is not concluded. It is usually a cut short process that leads to a limited or non-existent use of the software in the classroom.

In view of the foregoing, we set ourselves the objective of establishing the effects of a geometry teaching program used by the GeoGebra software on the geometry learning of secondary school students expressed in three skills: reasoning and demonstration, mathematical communication and problem-solving. The question to be answered was: What are the effects of an education using the GeoGebra software on the learning of geometry in the students of the 4th year of secondary school of a private educational institution in Lima? The three substantive hypotheses that guided the search for this answer were based on the assumption that the use of the software, as part of a larger geometry learning program, would significantly improve the previously mentioned students' skills. An additional hypothesis was that these improvements would be greater than those achievable with any other intervention.

GeoGebra Educational Software

GeoGebra multi-platform software (Hohenwarter & Preiner, 2007) combines the ease of use of other dynamic geometry software with the flexible possibilities of the algebraic software. GeoGebra's basic idea is to combine geometry, algebra and calculus, which other packages address separately, into a single package that can be used for teaching geometry from the elementary to the university level. The teaching of school geometry from a dynamic perspective is a relatively new field in teaching, but with an increasingly frequent and relevant presence (Ferreira, et al, 2009; Duval, 2000). For this reason, the National Council of Teachers of Mathematics (NCTM) promotes the use of dynamic geometry software in classrooms, including GeoGebra.

According to Sanchez (2003), GeoGebra has a set of features (table 1) that are especially appropriate for strengthening students' mathematical skills.

Table 1.

GeoGebra Attributes and Features.

Attributes	Features
Constructability	Possibility of building new scenarios based on the combination of objects in space and time. This concept is closely linked to the constructivist learning model.
Navigability	Free and flexible exploring, unlike other packages using fixed, linear and sequential routes.
Interactivity	System that provides the user with real-time feedback, in addition to dynamically adapting or modifying its performance according to the events and information received.
Quality of the Content	Reliability, relevance, organization and accessibility of the information contained in the software, which can additionally be adapted to different types of audiences.
Interface	Screen with which the trainees interacts, which captures the trainees' attention, guides their actions and reflects the state of the system.

In the specific field of geometry teaching, studies on interventions that used the GeoGebra software with teachers, who had been previously trained in its use, report that these teachers perceived that the package facilitated an interactive learning environment (Ferreira, et al, 2009), or learning based on collaborative processes (García, 2011). Other studies carried out within the context of the teachers' training in the use of GeoGebra report that teachers had positive opinions about its use in the teaching process, provided that it is based on adequate training (Tatar & Yilmaz, 2016; Bulut & Bulut, 2011). Also, studies conducted on students found improvements on dimensions such as the speed of activities or finding answers, the understanding of concepts, and the focus on certain topics or the motivation to study them in depth (Mendes, et al., 2014).

Geometry Learning

Geometry is considered as a reflexive field that allows to solve problems of diverse nature and to understand a world that offers a wide range of forms, be

it in natural or artificial scenarios (Gamboa & Ballesterero, 2009). It is also a strategic support in the professional training of any field due to its application in various contexts and scenarios, as well as the role it plays in strengthening logical reasoning (Báez & Iglesias, 2007).

According to Duval (1998), learning geometry involves at least three cognitive activities: construction, which refers to the design of configurations through geometric instruments; reasoning, linked to discursive and argumentative processes; and visualization, which focuses on spatial representations. Jones (2002) goes to a higher level of disaggregation by identifying the skills that geometry helps to develop in students: visualization, critical thinking, anticipation, problem-solving, hypothesis formulation, deductive reasoning, and logical argumentation in tests or demonstration processes.

The pedagogical work guidelines given by the Ministry of Education of Peru (MINEDU) in the National Curricular Design of Regular Basic Education (2009) in the area of mathematics, gather these elements and include them in a progression logic that should help the student reach some skills that change and increase their complexity in each cycle. The achievement of each of the four skills in the area of mathematics in the VII cycle (3rd, 4th, and 5th grade of secondary school) requires the development of skills, knowledge, and attitudes, which are worked on in the so-called transversal processes. The Geometry Skill is: “To solve problematic situations of real and mathematical context that imply the use of properties and geometric relations in construction and movement, in the plane and space, using diverse strategies of solution and justifying their procedures and results” (MINEDU, 2014: pp, 6). The transversal skills are: reasoning and demonstration, mathematical communication, and problem-solving.

According to the Learning Paths Guide (MINEDU, 2014), the reasoning and demonstration capacity consists of arguing which generates mathematical ideas, justifying and validating conclusions, assumptions, conjectures and hypotheses, supported by meanings and properties of numbers and operations.

The ability to communicate mathematically consists of expressing, in written or verbal form, mathematical ideas, the meaning of numbers, operations or other processes, making use of different representations and a simple mathematical language. The ability to solve problems consists of developing and using strategies, planning, executing and evaluating heuristic strategies, calculation procedures, comparison or estimation, using various resources. The latter skill is essential and its activation must take place within the framework of problems whose proximity to the students environment motivates them to commit to its solution, starting from the assumption that the paths towards the answer are diverse and therefore, the students must actively participate in the process by connecting with previous knowledge or by risking to offer new proposals in situations where creativity must play a key role (MINEDU, 2007).

Intervention Program Using the Geogebra Software

The intervention program was based on the teaching of a dynamic geometry with the use of the GeoGebra software. It was included in the Geometry course of the 4th year of secondary school (Mathematics of the VII cycle). The program's purpose was to strengthen the students' reasoning and demonstration skills, mathematical communication, and problem-solving, while evaluating such process. We worked with a skills-based approach and with teaching methods that emphasized problem-solving. Both teaching orientations are part of the recommendations given by the MINEDU as part of the National Curricular Design of Regular Basic Education (2009) in the area of mathematics, and explicitly appear as part of the secondary-level curriculum of the institution in which the intervention was carried out. Therefore, the only difference between the intervention and the regular program was the use of the GeoGebra software.

The intervention program consisted of 11 sessions of three hours each, one hour for the presentation of the necessary geometric concepts and principles, the ways of approaching the problems, and the expected results;

and two hours to solve the problems using the software. It took a total of two months.

In each session (of two hours each), the software was used in groups of two students. In general, the emphasis during the 11 sessions was on the use of methodologies and didactics based on the problematization. The teacher did not offer previously elaborated knowledge but rather addressed specific topics or situations, from which the teacher helped the students to identify deficiencies or contradictions, in order to give them the nature of problems. These problems were the core around which the whole learning process was organized. In operational terms, the teacher offered one or more problems that challenged students to develop their skills in order to explain or propose solutions. To do this, students had to search for, identify and select the information they needed, use it to find possible solutions to the problems, and communicate them using the parameters recognized by geometry.

The GeoGebra features, which is user-friendly software for visual or numerical verification or demonstration of theorems and properties, facilitated the problematization and resolution of problems. During the activities designed for each session, where problem-solving process was done from the software environment, the students had the opportunity to discover for themselves. The teacher just carried out an effective monitoring by means of questions and suggestions, without imposing the solutions which helped the students to elaborate their own solution plans and to discover the results. This way of working was aligned with Polya's pedagogy (1995).

Methodology

An analytical and observational study was conducted to evaluate the changes in reasoning and demonstration skills, mathematical communication, and problem-solving skills experienced by 4th year secondary school students at a private educational institution in Lima, within the framework of geometry learning. The students were monitored in two groups. The experimental group exposed to the Geometry course of the 4th year of secondary school

(Mathematics of the VII cycle) with the use of the GeoGebra software for a dynamic geometry teaching. The control group was also exposed to the Geometry course of the 4th year of secondary school (Mathematics of the VII cycle) but with the traditional teaching, without using the software.

Participants

The groups were made up of 24 students each, with ages ranging from 15 to 16 years, without cases of over-age. Of the 48 students, 40% were men and 60% were women. The groups were assigned based on pre-established classrooms according to the enrolment processes of the educational institution, which is assigned randomly. There were groups (classrooms) similar in their performance averages, ages, and gender proportions.

Instruments and Techniques

The Geometry Learning Assessment Test was an ad hoc development for the purposes of this study. It was an inventory composed of 10 items to evaluate the three dimensions observed: reasoning and demonstration with three items, mathematical communication with three items, and problem-solving with four items. The test used a grading system of 0/20.

Validity based on content (representativeness and relevance) was assessed by experts' judgment, whose process resulted in Aiken's V values greater than .75 for all items. Reliability was tested in a sample of similar characteristics to the group of students with the use of the test-retest method applied with an interval of 60 days. Correlation coefficients higher than .80 were obtained for the three dimensions. This same sample served to establish the internal consistency of the instrument through the alpha coefficient, which reached values of .822 for reasoning and demonstration, .867 for mathematical communication, and .850 for problem-solving.

Procedures

The evaluation of the three skills used data obtained from the Geometry Learning Assessment Test that was applied to the experimental group and the control group *before* and *after* the intervention. The application of the *pre-test* confirmed the equivalence of the performance of both groups in the evaluated skills. Then came the intervention, with 11 sessions of three hours each, which took place in two months. Finally, the *post-test* application established the differences in the mentioned performances between both groups.

The methodological design used included two types of analyses: comparison of scores by moments within each group (intragroup analysis) and by moments between the two groups (intergroup analysis). Due to the sample size and sampling characteristics, the *Wilcoxon T test* was used for the intragroup analysis, while the *Mann Whitney U test* was used for the intergroup analysis, and then the magnitude of the effect on the biserial correlation (r_b) form was calculated (Fritz, Morris & Richler, 2012). The following was considered in order to differentiate the groups: a magnitude of the r_b : $< .10$ as no significant; between $.10$ and $.30$, low, between $.30$ and $.50$, moderate; and greater than $.50$, high.

Results

Table 2.

Differences of related groups in assessed geometric skills

Dimension		Pre-test		Post-test		Z	p	r_b
		M	SD	M	SD			
Mathematical communication	Experimental G.	2.41	.94	4.27	1.26	-3.89	.000**	.56
	Control G.	2.45	1.33	3.45	1.19	-3.07	.002**	.44
Reasoning and demonstration	Experimental G.	1.58	.77	2.62	.78	-3.37	.001**	.49
	Control G.	1.66	.88	2.14	.71	-2.33	.019*	.34
Problem- solving	Experimental G.	2.66	1.07	4.89	1.75	-3.64	.000**	.53
	Control G.	2.58	.90	3.77	.84	-3.67	.000**	.53

*p < .05

**p < .01

At the intergroup level (table 1) in both groups, the evaluation of reasoning and demonstration skills, mathematical communication, and problem-solving, was always favorable in the *post-test*, with statistically significant differences. On the complementary analysis to establish the practical significance of the differences, only the experimental group reached high levels ($r_b > .50$), while the control group showed moderate levels ($r_b > .30$).

Table 3.

Independent group differences in assessed geometric skills

Dimension		Experimental Group		Control Group		U	p	r_b
		M	SD	M	SD			
Mathematical communication	Pre-test	2.41	.94	2.45	1.33	264.50	.623	.07
	Post-test	4.27	1.26	3.45	1.19	179.50	.024*	.33
Reasoning and demonstration	Pre-test	1.58	.77	1.66	.88	265.50	.636	.06
	Post-test	2.62	.78	2.14	.71	190.00	.039*	.30
Problem- solving	Pre-test	2.66	1.07	2.58	.90	262.00	.586	.07
	Post-test	4.89	1.75	3.77	.84	173.00	.016*	.35

* $p < .05$

At the intergroup level (table 2), the *pre-test* showed similar scores in both groups for the three skills. In the *post-test*, statistically significant differences were found between the groups ($p < .05$) in all three skills, always favorable to the target group. In addition, it was established that these differences reach only a moderate level in all cases ($r_b > .30$).

Discussion

The results allowed us to achieve the expected objective, since the use of a software such as GeoGebra, in the framework of a geometry course in a Secondary Education School, had important effects on the students' learning process, as regards to the strengthening of their reasoning and demonstration, mathematical communication, and problem- solving skills.

In addition, the results suggest additional effects such as the facilitating group work, collaborative learning processes, and favorable opinions on the use of software from teachers.

The scores obtained in the application of the Assessment Test of Learning in Geometry were higher in both groups in the *post-test* on the three evaluated skills, which showed that any intervention has positive effects. However, these results showed certain trends. Between the *pre-test* and the *post-test*, the experimental group reached statistically significant differences at high levels ($r_{bis} > .50$), while the control group reached only moderate levels ($r_{bis} > .30$). In the *post-test*, after the parity observed in the *pre-test*, the experimental group reached scores higher than those of the control group, with significant differences at moderate levels ($r_{bis} > .30$). The intervention with the use of the GeoGebra software showed that it produces more significant changes than traditional teaching.

These results coincide with those reported by other studies on the effects of the use of ICT or, specifically, of the GeoGebra software in the classroom. De la Chica (2010), found that the use of ICT leads students to achieve improvements in intellectual skills such as reasoning and problem-solving. Mendes, et al., (2014) found that the use of GeoGebra makes it possible for students to improve their speed in performing activities or getting answers, as well as to understand concepts.

It is logical to infer that effects such as those reported were produced because there were certain favorable conditions for this to occur. One of these conditions had to do with the fact that the teacher in charge of the intervention had the required skills to integrate the GeoGebra software as part of the teaching process, bringing its real use closer to the potential of the software itself. This is an indispensable condition outlined in a previous study by Parsad, Lewis & Farris (2001), who stated that the teachers' technological skills are a key factor for integrating ICT into a good teaching context. In this sense, an integral technological training is strategic so that a software like

GeoGebra becomes a productive tool and, what is more important, teachers consider it as such (Tatar & Yilmaz, 2016; Bulut & Bulut, 2011).

Another favorable condition was that the educational institution, where the intervention was carried out, has as a pedagogical orientation learning environment where groups work to address issues or solve problems. Students are used to studying that way. In this context, the work-in-pairs system was not only facilitated but progressively turned into collaborative work where skills were complemented, with an important effect on how and how much of the assigned tasks could be performed. The pairs contributed decisively to the overcoming of obstacles, or at least to the perception that the difficulties were less than if they had carried out the work alone. This finding is consistent with that reported by other studies that used GeoGebra in the field of mathematics and found collaborative learning (García, 2011; Lavy & Leron, 2004) and emergent interaction (Ferreira, et al, 2009) environments.

In the opinion of the teacher in charge of the control groups, GeoGebra turned out to be a user-friendly program for the students, which required little time to familiarize themselves with the software's resources. The teacher also said that the use of the software made it easier for students to understand difficult concepts, helping them to overcome some obstacles of this teaching field. This opinion is consistent with those reported in other studies (Mendes, et al., 2014; Ferreira, et al., 2009).

As conclusions, we can say that the four hypotheses formulated were validated. The intragroup analysis reported that in all cases (reasoning and demonstration, mathematical communication, and problem-solving skills) the scores were higher at the *post-test*, with differences that were significant at high levels ($r_b > .50$). The intergroup analysis reported that the scores reached *post-test* were favorable in all cases to the experimental group, with differences that were significant at moderate levels ($r_b > .30$).

Finally, this experience leads us to conclude that the research on the teaching practice can have important implications for the teaching process,

providing favorable evidence for the use of strategies or tools, or simply providing ideas that can give teachers the opportunity to use them to improve processes. Unlike research with fundamentally cognitive purposes, which seeks the production of generalizable knowledge and, therefore, difficult to apply to the particular conditions of the daily practice of a specific teacher, the applied research generates knowledge from common situations, which are relevant and easy to approach.

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